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Technical Note

Numerical analysis of double-diffusive convection in a porous enclosure due to opposing heat and mass fluxes on the vertical walls – Why does peculiar oscillation occur?

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Abstract

Peculiar oscillating convection is observed when two-dimensional double-diffusive convection in porous medium is analysed numerically. The top and bottom walls of an enclosure are insulated, and constant and opposing heat and mass fluxes are prescribed on the vertical walls. The peculiar oscillations are of three types: (1) Chaotic oscillations wherein the main flow is due to temperature; however, the convection due to concentration is strong enough to generate this peculiar oscillation. (2) The 'sudden steady state case' caused by the shifts from thermally-driven to concentration-driven forces. (3) The 're-oscillation case' caused by the convection pattern changes from centrosymmetric to non-centrosymmetric.

Keywords: Double diffusion; Porous medium; Buoyancy ratio; Numerical simulation; Peculiar oscillation

1. Introduction

Double-diffusive convection in porous medium, which occurs because of temperature and concentration differences, is observed in many disciplines, for example, electrochemistry, geophysics, etc. [1–3]. Because heat and mass transfers in a membrane influence the reaction [4], it is important to understand the double-diffusive convection in porous media in detail. Various authors have theoretically and numerically studied the double-diffusive convection in a fluid-saturated porous enclosure due to the opposing heat and mass fluxes on vertical walls [5–12]. In these studies, the numerical calculations yielded oscillatory solutions [9,10]. In the former paper [9], we performed calculations only when the aspect ratio was 5. Furthermore, it has been observed that the oscillation pattern of Nusselt number Nu changes abruptly with time. In this paper, we have performed calculations in order to clarify why such peculiar oscillations occur. By analysing the double-diffusive convection pattern, we intend to elucidate the physical mechanism responsible for the occurrence of such oscillations. In conclusion, three distinct kinds of peculiar oscillations can be observed, and accordingly, the peculiar oscillations are classified into three types.

2. Problem statements

We consider a two-dimensional vertical enclosure filled with a homogeneous, fluid-saturated porous medium of aspect ratio A. The top and bottom walls are insulated. Constant heat flux Λ_T and mass flux Λ_c are prescribed through the vertical walls. The governing equations are as follows: equation of momentum conservation in the Darcy regime with the Boussinesq approximation, equation of continuity and equations for the mass and thermal

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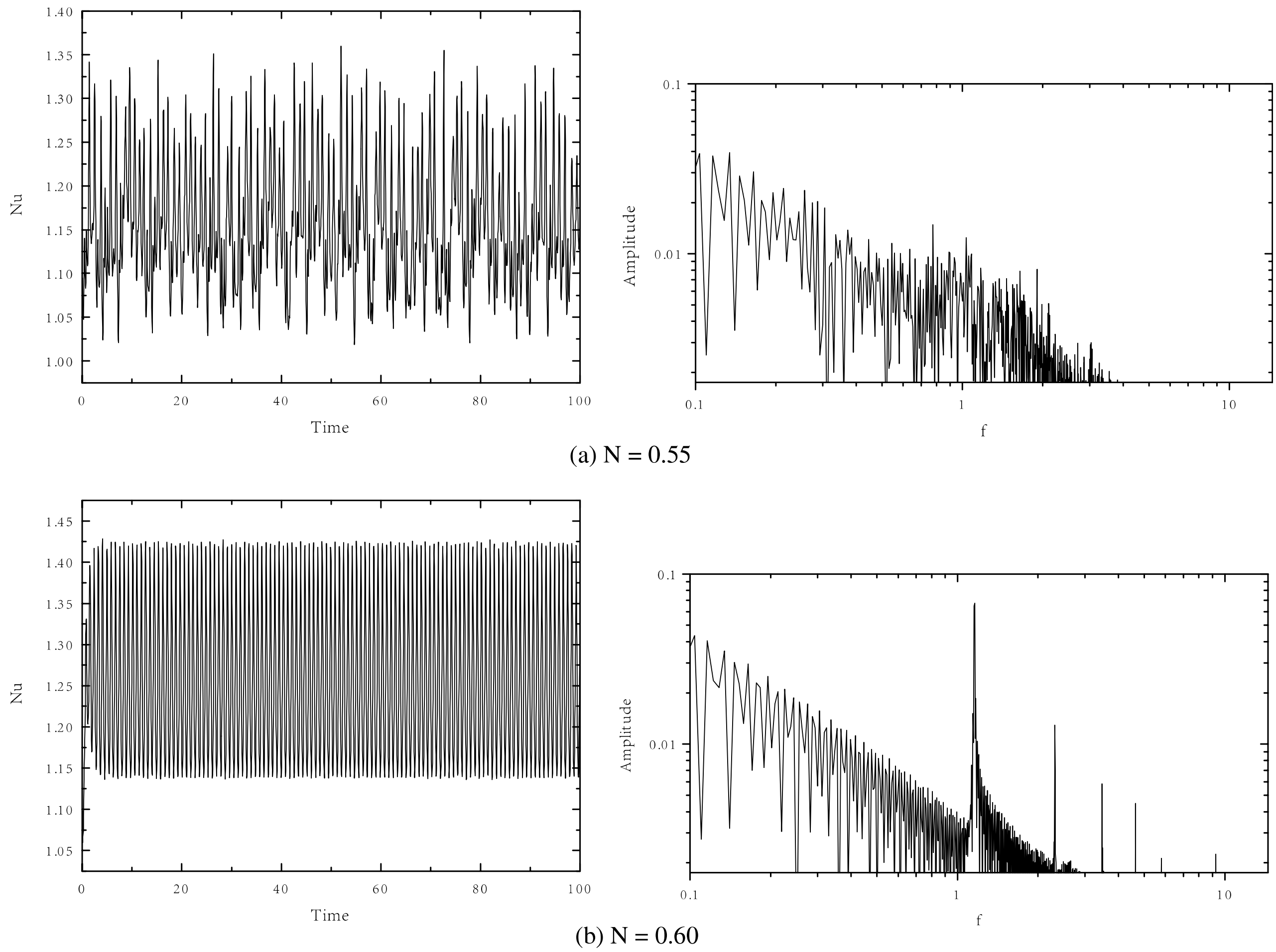


(a) N = 0.55

(b) N = 0.60

Fig. 1. FFT of the time-dependence of Nusselt number when $R = 100$, $Le = 20$ and $A = 5$.

energy conservation. The velocities, temperature and concentration are zero at the initial condition. The buoyancy ratio is defined by

$$N = \frac{\alpha \Lambda_T}{\beta \Lambda_c}, \tag{1}$$

where α is coefficient of thermal expansion and β is coefficient of concentration expansion.

Governing equations are solved numerically with the boundary and initial conditions by the finite difference method. No grid point is set on the physical boundaries ($|x| = 1$ and $|y| = A$). The first and end grid points are placed at a distance of half a grid space away from the boundaries. The boundary conditions at the walls are applied to these points. The numerical scheme used here is second-order accurate in space and first-order accurate in time. The matrices are solved under the given boundary conditions by the conjugate gradient method. For further details regarding this method, refer to Ref. [8].

In the previous calculations [9], the numerical grids of 62×302 were sufficient because we considered only the simple oscillation case. However, it is necessary to use a smaller mesh in order to study the peculiar oscillation problem. We have attempted to calculate when the grid size exceeds 62×302. The oscillation patterns of Nu are similar when the grid size exceeds 102×502. Therefore, we can arrive at a solution of sufficient accuracy in the present calculation if we use grids greater than 102×502.

In the present study, we performed calculations for the following cases: the Rayleigh–Darcy number $R = 50$, 100 and 200; the Lewis number $Le = 2$, 5, 10, 20 and 50; and the aspect ratio $A = 2.5$ and 5.

3. Results and discussion

3.1. Chaotic oscillation case

N_{min} is defined as the minimum value of the buoyancy ratio that generates oscillation. In our previous study [9], we observed that the oscillation of Nu was very complex near N_{min}; however, the reason for this oscillation pattern was not clear. Fig. 1 shows the oscillations of Nu and the corresponding FFT when $R = 100$, $Le = 20$ and $A = 5$. In this case, $N_{min} = 0.53$. As shown in Fig. 1a, Nu oscillates randomly and a clear peak is not observed in the FFT. From these figures, chaotic oscillation can be observed when $N = 0.55$. In the previous research [9], we were unable to determine whether the oscillation of Nu was chaotic or not. The oscillation is certainly chaotic in the present calculation when N is near $N_{\min}$. With a slight increase in N, the complex oscillation pattern changes to a simple one. In Fig. 1b, $N = 0.60$. The oscillation appears monotonous and a prominent peak is observed in the FFT. We also need to elucidate the convection pattern of chaotic oscillation. Fig. 2 shows the contour lines of stream functions when $R = 100$, $Le = 20$ and $N = 0.55$. In these stream functions, positive values correspond to the clockwise flow caused by temperature gradients and negative values to the counter-clockwise flow caused by concentration gradients. Fig. 2 reveals that the main flow is driven thermally. However, the convection due to the concentration gradients exists, and the magnitude of velocity and the convection pattern change periodically. Sometimes, the flow pattern divides the convection cell into an upper and a lower zone. In this case, the intensity of the convection becomes weak and as a result Nu assumes small values. When a single convection cell is formed and the flow is thermally-driven, Nu assumes large values. In addition, the convection patterns are centrosymmetric.

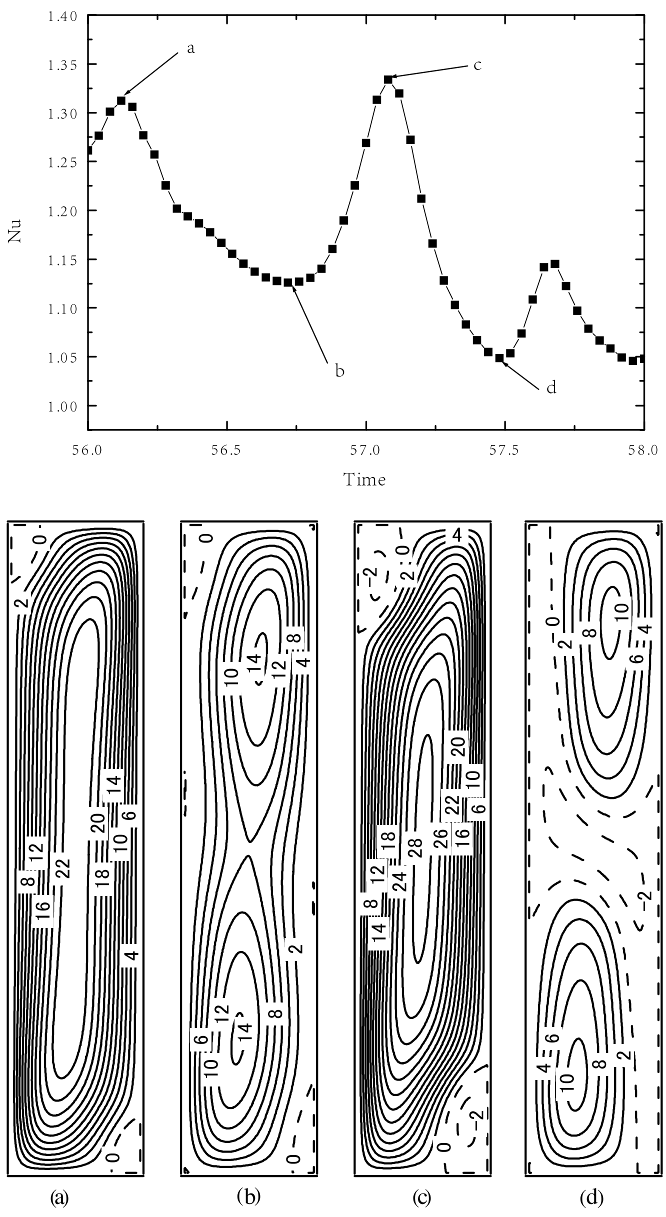
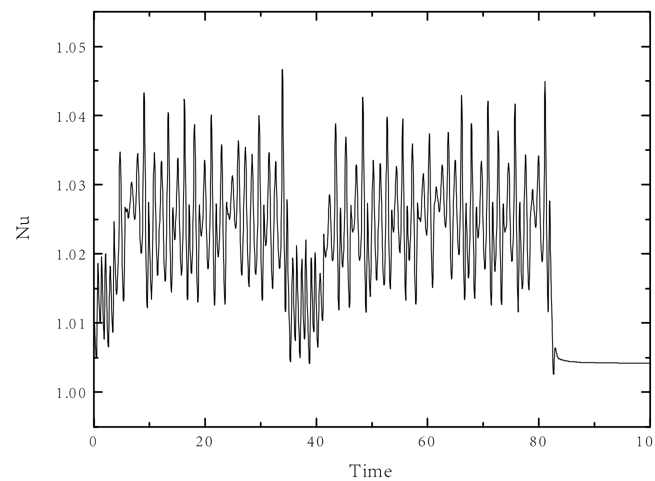


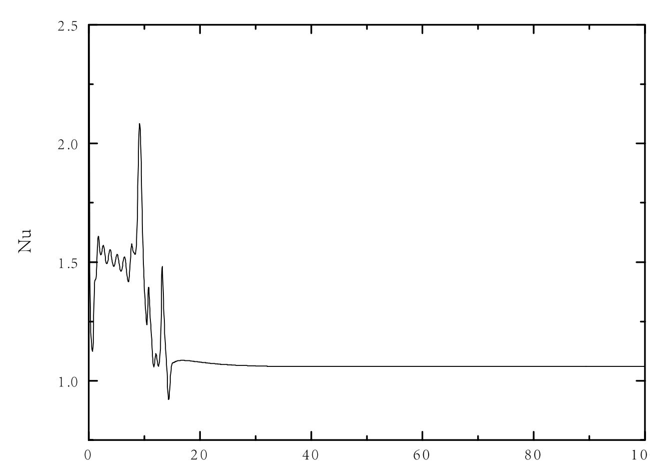
Fig. 2. Nu and contour lines of stream functions for $R = 100$, $Le = 20$, $N = 0.55$ and $A = 5$.

3.2. Sudden steady state case

From other calculations, we sometimes observe cases where the oscillation stops suddenly and the system reaches a steady state, as shown in Fig. 3. Such a pattern is referred to as 'sudden steady state case' in the present paper. Parametric values for Fig. 3a are $R = 50$, $Le = 50$, $A = 5$ and $N = 0.61$, and those for Fig. 3b are $R = 100$, $Le = 2$, $A = 5$ and $N = 0.83$. Based on these figures, the 'sudden steady state case' can be described as follows: Initially, the oscillation is chaotic. In the middle of the chaotic period, Nu drops abruptly to a very small value; then, it rises slightly and attains the steady state. Such peculiar pattern



(a) R = 50, Le = 50, N = 0.61 and A = 5



(b) R = 100, Le = 2, N = 0.83 and A = 5

Fig. 3. Sudden steady state case of Nu.

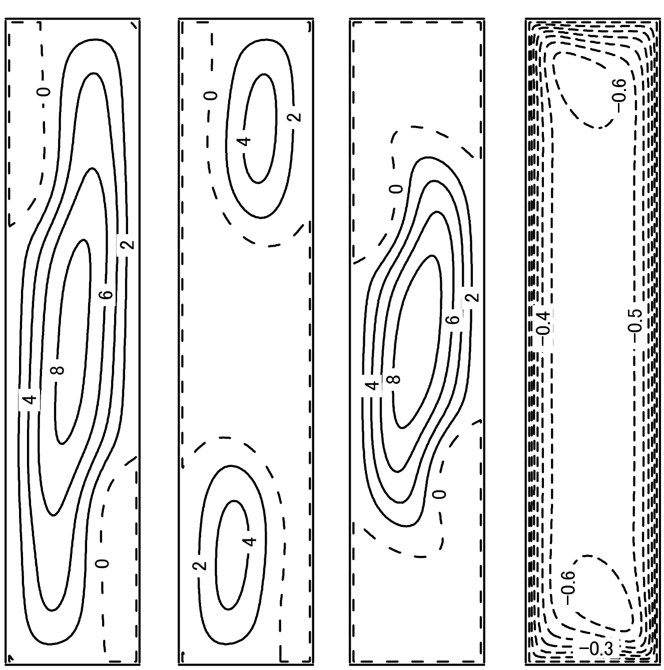
can only be observed when N is slightly smaller than $N_{\min}$ and $A \geqslant 5$.

Fig. 4 shows the contour lines of the stream functions of the 'sudden steady state case' when $R = 100$, $Le = 2$, $A = 5$ and $N = 0.83$ (Fig. 3b). Until t is 13.2, the chaotic oscillation persists. The oscillation pattern is similar to the chaotic oscillation case shown in Fig. 2. Initially, the convection is temperature-dominated, and the convection cell is divided into upper and lower zones. However, in the steady state, the direction of the convection is reversed. Eventually, the convection is driven completely by the concentration gradients (counter-clockwise flow). Therefore, in the 'sudden steady state case', the chaotic oscillation occurs because the temperature-dominated and concentration-dominated convections compete against each other in the beginning. The concentration-driven force evolves gradually with time and eventually dominates the entire domain. Then, the system reaches a steady state. In the present case, it should be noted that the buoyancy ratio N is less than $N_{\min}$ and outside the oscillation region. The reason for this peculiar phenomenon is probably the different diffusivities of the concentration and the thermal energy. It is observed that a longer time is required to attain the steady state when a large Le is considered ($Le = 50$) than when $Le = 2$.

3.3. Re-oscillation case

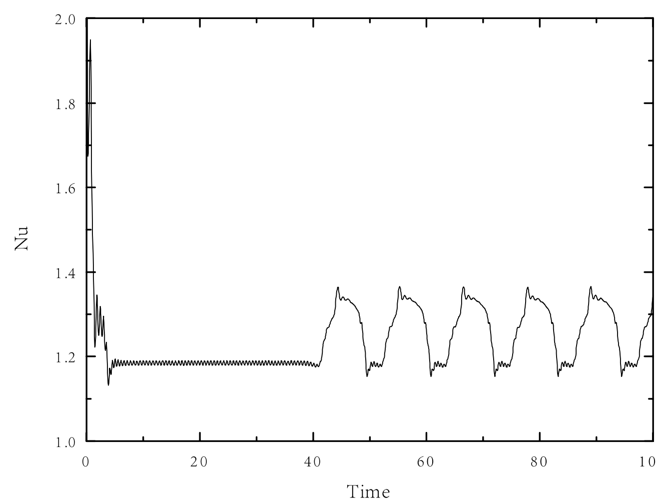
We think that this is the most peculiar case of oscillation we have ever seen. This phenomenon can be only observed when R is large, N is slightly larger than $N_{\min}$, and A is 2.5; it is a very uncommon situation. Fig. 5a shows Nu as a function of time when $R = 200$, $Le = 2$, $A = 2.5$ and $N = 0.83$, and Fig. 5b shows Nu as a function of time when $R = 200$, $Le = 10$, $A = 2.5$ and $N = 0.56$. From both figures, it appears that the oscillation is damped at an early stage and maintains a steady state until about $t < 40$. After that, however, Nu starts oscillating and this re-oscillation has a very large period.

The flow pattern for the re-oscillation case is shown in Fig. 6. In the first case shown in Fig. 6a, the oscillation is just damping. The main flow is convection due to temperature gradients but there are convection cells due to concentration gradients on the left and right hand sides. After that, in (b), (c) and (d), the convection due to the temperature becomes weaker and that due to the concentration creates a main flow path along the boundary surrounding a thermally-driven cell. The convection pattern is centrosymmetric in these four cases [(a)–(d)]; however, it becomes non-centrosymmetric with time. In (e), the con-

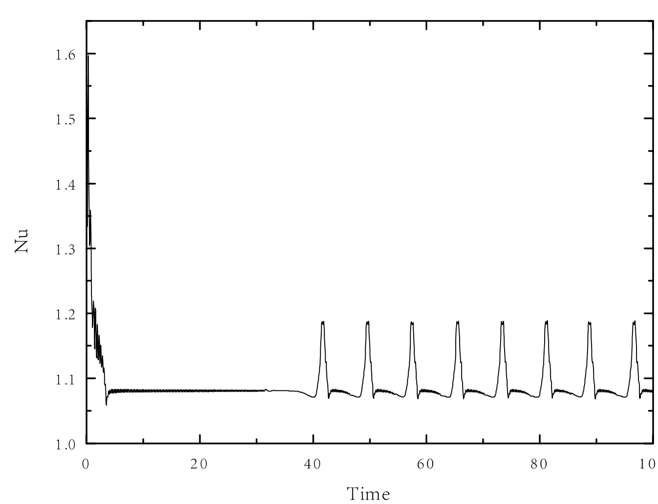


Nu = 2.084	1.235	1.482	1.060
t = 9.08	10.4	13.2	100

Fig. 4. Contour lines of the stream functions of the sudden steady state case for $R = 100$, $Le = 2$, $A = 5$ and $N = 0.83$.



(a) R = 200, Le = 2, N = 0.83 and A = 2.5



(b) R = 200, Le = 10, N = 0.56 and A = 2.5

Fig. 5. Re-oscillation case of Nu.

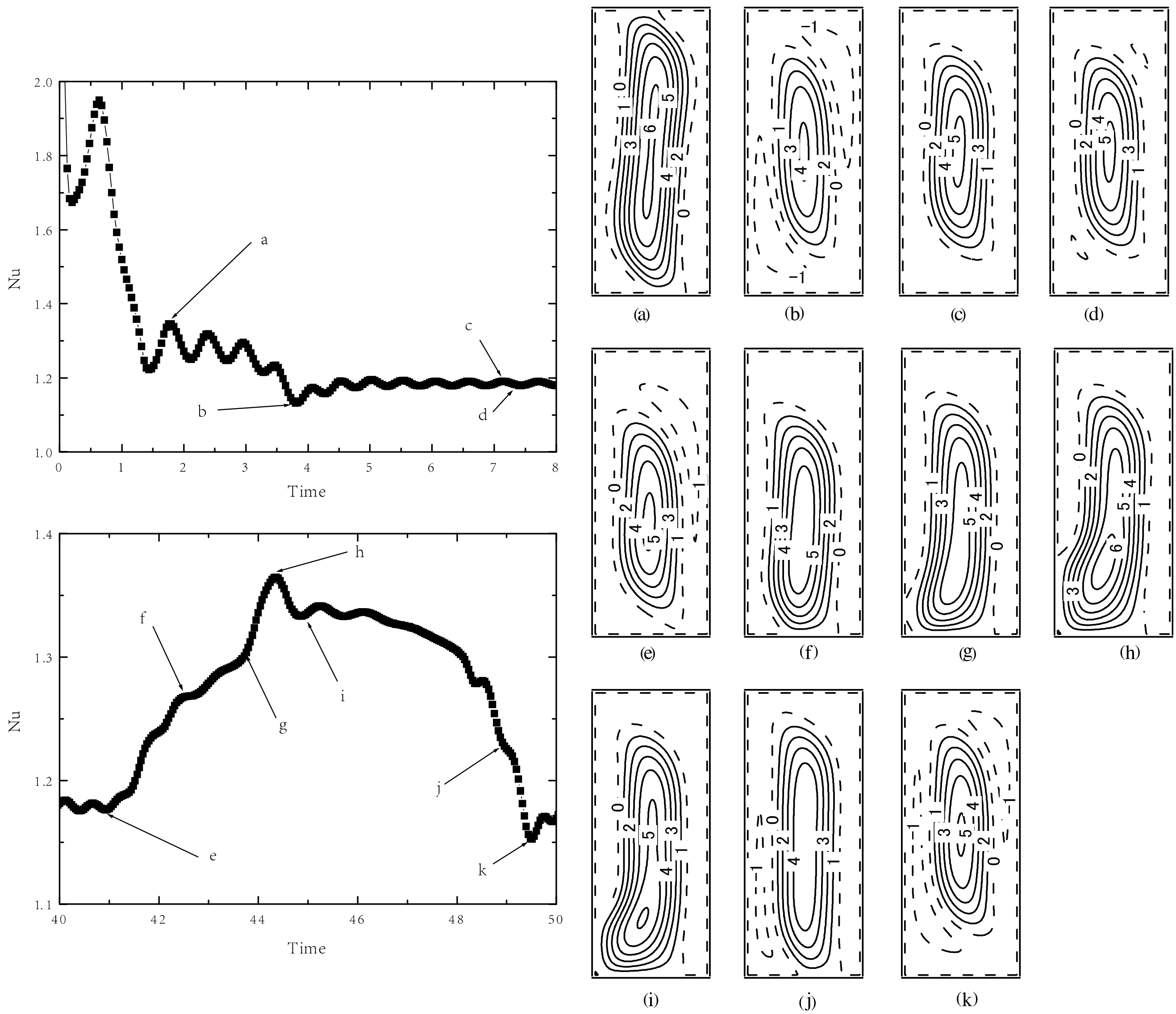


Fig. 6. Nu and contour lines of the stream functions for $R = 200$, Le = 2, $N = 0.83$ and $A = 2.5$.

vection cell due to the temperature moves downward and thereafter Nu increases slightly. Subsequently, this convection cell becomes distorted, particularly in the lower and left region, as can be observed in (g) and (h). Then, the convection due to the temperature becomes stronger and Nu increases and attains the peak value at (h). The flow becomes weak and Nu decreases; however, the deformed convection pattern still retains its shape as shown in (i). Thereafter, the flow pattern gradually restores to the centrosymmetric pattern. Since such flow pattern changes require a very long time, the Nu re-oscillation has a long period.

A possible situation is discussed below. By the time $t = 5$, the concentration has diffused in the entire domain, which brings the system to a quasi-stable steady state. However, it starts readjusting itself to a more stable state, and this process continues up to $t \cong 40$. Eventually, the system attains a slowly oscillating convection as the most stable state.

4. Conclusions

The three types of peculiar oscillations—chaotic oscillation, sudden steady state case and re-oscillation—have been investigated and the conditions for generating them have been explained. Chaotic oscillation is generated when buoyancy ratio N is near $N_{\min}$. Though the main flow is due to temperature, convection due to concentration is strong enough to generate this peculiar oscillation. When N is slightly less than $N_{\min}$ and $A \geqslant 5$, sometimes the 'sudden steady state case' can be observed. This phenomenon is caused by the shift from the thermally-driven force to the concentration-driven force. The 're-oscillation case' can be observed only when N is slightly greater than $N_{\min}$ and $A = 2.5$. This phenomenon occurs because the convection pattern changes from centrosymmetric to non-centrosymmetric. Since this change requires a very long time, the re-oscillation typically has a very long period.

Such phenomena appear unusual; however, they are understandable in terms of the convection-cell-pattern movement. Perhaps we have ignored such phenomena when we glimpsed them. Because double-diffusive convection in porous media can be observed in many fields, there is a possibility of discovering the phenomena wherein the state of oscillation changes significantly with time.

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